A B S T R A C T

A two-stage mixed integer linear programming model (MILP) incorporating a novel method of stochastic scenario generation was proposed in order to optimize the economic performance of the synergistic combination of midstream and downstream petrochemical supply chain. The uncertainty nature of the problem intrigued the parameter estimation, which was conducted through discretizing the assumed probability distribution of the stochastic parameters. The modeling framework was adapted into a real-world scale of petrochemical enterprise and fed into optimization computations. Comparisons between the deterministic model and stochastic model were discussed, and the influences of the cost components on the overall profit were analyzed. The computational results demonstrated the rationality of using reasonable numbers of scenarios to approximate the stochastic optimization problem.

1. Introduction

It is through supply chains that raw materials are acquired, stored, and transformed into products, which are then delivered to customers. Information, materials and financial flows occur among different echelons over the time horizon of the supply chain [1]. The petrochemical industry is often considered as three major segments: upstream, midstream, and downstream [2]. The midstream and downstream segments cover the crude oil transformation, production of petrochemical products, inventory and distribution of end products to consumers. As a complex and dynamic system, the petrochemical supply chain is subjected to uncertainties involved in the entire procedures. Most often, the uncertain factors involved in the petrochemical supply chain are fluctuations of production levels, product prices and unforeseeable demands of the products [3]. For the deterministic models, those early works on planning and scheduling of petrochemical supply chains [4-6] cannot capture the uncertain feature of most real-world applications. The perturbations of uncertain factors may cause potentially significant impacts on the overall financial outcomes, characterizing the dynamic behaviors of the large-scale petrochemical industry, especially when one considers it as a capital intense infrastructure. In order to maintain the operation profitability, the aforementioned uncertainty factors must be taken into consideration.

To deal with uncertainty factors in the optimization problem, different approaches employing robust optimization [7,8], chance constraint optimization [9,10], fuzzy programming [11,12], and stochastic programming [13-15] have been proposed. The stochastic programming methods were shown to exhibit good performance when the collected data have a particular distribution [16]. In addition, Lima et al. [17], AlOthman et al. [18], and Leiras et al. [19] showed that discretizing the normal distribution of the values of the uncertain parameters into different scenarios would be computationally more tractable and industrially more applicable than handling the continuous random parameters involved in the stochastic programming methods.

In spite of a considerable amount of research focusing on scenariobased stochastic programming methods to deal with the uncertainty of the petrochemical supply chain [18-20], most articles refer to the segmentation of the scenarios only with limited number of scenarios. One of the first works addressing the scenario discretization was by Al-Othman et al. [18], whose work dealt with the uncertainty of market demand and price based on the two-stage problem with finite numbers of scenarios. Leiras et al. [19] proposed a simplified scenario tree with 9 scenarios based on data collected from industrial experts and from secondary research like historical economic data. Zhao et al. [20] followed the similar approach and the independent uncertain parameters (product price and demand) were measured with three levels of realizations (high, medium and low, and the 3 realizations were assigned with probabilities of and , respectively), so 9 scenarios were generated.

However, limited numbers of scenarios proposed in his work did not capture the uncertainty of the problem and calculation inaccuracy cannot be ignored. If this method were adopted to discretize the stochastic parameters into more refined grids of scenarios to embed more possible incidents, the computational burden would be a significant issue. To the best of our knowledge, few previous works have done any explorations on the midpoint between the calculation accuracy and computational burden.

Based on the shortcomings and gaps identified from the literature review, the contributions of this paper can be summarized as follows: the first novelty of this work is that it highlights the relationship between scenario generation and the probability distribution of the uncertain parameters, which can make the scenario generation more rational and can avoid the arbitrary assigning of probability to each scenario. The second novelty is that this work analyzed the trade-off between calculation accuracy and computational time and proposed a criterion for choosing the midpoint. The third novelty is that this work analyzed the influence of each cost component on the overall profit of the supply chain model.

The remainder of the paper is organized as follows. In Section 2 the problem is stated, and assumptions are offered for simplification of the subsequent model. Section 3 is dedicated to the formulation of a scenario-based two stage stochastic optimization model. In Section 4, a case study in the context of large-scale petrochemical supply chain industry is conducted, more scenarios are generated in this section, and influences of the cost components on the overall profit were analyzed. Conclusions are elaborated in Section 5, and some final remarks close the paper.

**2. Problem Statement**

A superstructure of the petrochemical supply chain with four levels of participants: suppliers, production centers, distribution centers and consumers [19], is illustrated in Fig. 1.

The participants in the supply chain denoted by the nodes in the superstructure in Fig. 1 can be described as:

1. Both the oil field and the terminal (TE) are suppliers of raw materials. Meanwhile, the terminal functions as a receiver of final products that are to be exported.
2. The refineries ( ) function as warehouses for the raw materials and production centers transforming the raw materials to the products, which are then transported to distribution bases (B: b1, b2, b3) and/or the terminal.
3. Both the distribution bases and the terminal act as warehouses for the products.
4. Products stocked in distribution bases are targeted at domestic customers (C: c1, c2, c3, c4, c5), while products stocked in terminal are targeted at oversea customers (OC: oc1, oc2).

The number of nodes in the four levels can be adjusted according to different scales of the real cases.

When product demand of customers goes beyond the upper limit of refinery production, extra products need to be procured from overseas with possibly a higher price at the terminal to compensate for the existing demand. Extra products can be sold on site to the oversea customers or can be transported to the distribution bases to be sold to the domestic customers.

For the sake of simplification, the following assumptions are adopted in the present work:

1. The physical structure of the supply chain has been predefined and will have no alterations during the whole-time horizon.
2. The stochastic parameters are defined as the uncertain prices and demands of the products and can be represented as stochastic parameters with normal distribution of probabilities, the means and standard deviations of which are able to be forecasted via market data mining and analyses.
3. Parameters with respect to each node in the supply chain structure in Fig. 1, such as the prices and properties of each material, the yield ratio, properties and kinds of product produced by each refinery, the upper and lower bounds of each kind of capacity constraint, and the unit price of each kind of cost component, are given and will remain constant for each time period discussed.
4. Each node (including the refineries) in the supply chain is regarded as black box, with inner and outer flow of products and materials being monitored.
5. **Mathematical Model**

A two-stage stochastic linear programing method with resource was first proposed by Beale [21] and Danzig [22]. In their approach, some decisions, termed as resource actions, are made after uncertainty is disclosed, and the decision variables can thus be categorized according to stages [23]. Based on the two-stage model, Zhao et al. [20] developed a two-stage scenario-based mixed-integer linear programming (MILP) method for the optimization of petrochemical supply chain. In the following subsections, the model by Zhao is employed as a basis.

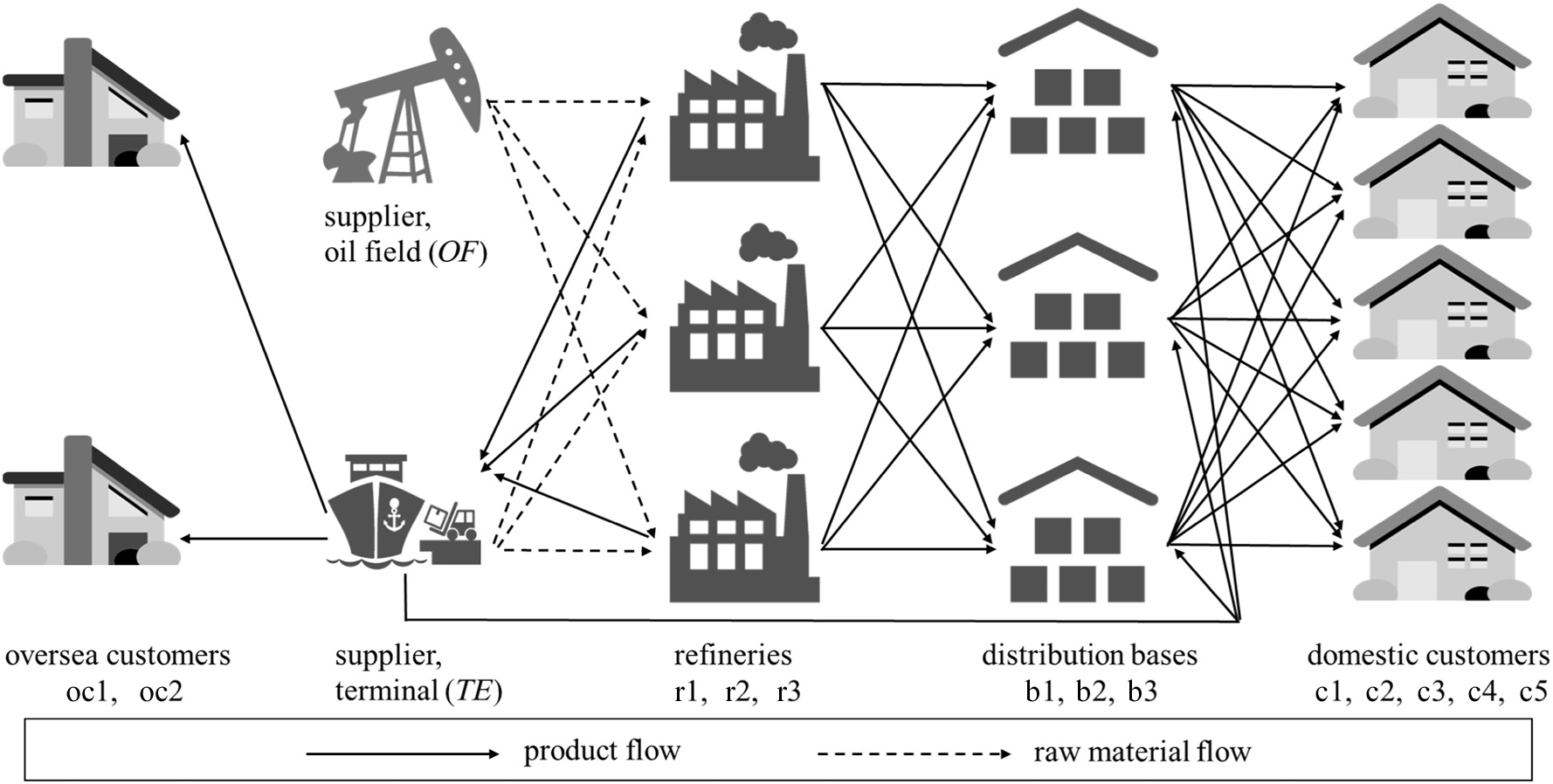


Fig. 1. Petroleum supply chain network.

The terms in the objective function are categorized into two stages. The cost of materials purchased at each supplier and the cost of materials transported between nodes that are involved in the supply chain are defined as the first stage terms and therefore scenario-independent, while the other terms are sorted as the second stage ones, which are scenariodependent.  
3.1. Objective function

The objective of the supply chain optimization is to maximize the expected profit, which follows the basic rule of sales subtract costs, as given by Eq. (1). The costs include those for material purchase (Cmp), material transportation (Cmtr), carbon emission tax (Cctax), refinery operation (Cro), inventory (Cin), product transportation (Cptr), extra product purchase (Cepp), production surplus penalty (Csur), and production backlog penalty (Cbck).

In the function given by Eq. (1), refers to 'scenario refers to the probability of scenario .

The sale is given by

where and refer to the unit price for product purchased at time period tp of scenario at respectively, terminal te and distribution , tp .p refer to the quantity of product sold at time period of scenario at respectively, terminal te to overseas customer and distribution to domestic customer .

The costs in Eq. (1) are as follows:

**The material purchase:**

where and refer to the unit prices of material purchased during time period at respectively terminal te and oil field and refer to quantity of material purchased by refinery during time period at respectively terminal te and oil field of.

**The material transportation:**

where refers to the unit price of material transportation for material by transportation tool at time period from node to node refers to the distance between nodes and ; and refers to quantity of material transported from node to by transportation tool at time period .

**The carbon emission tax:**

The environmental influence in the form of carbon emission tax (Cctax) was incorporated into the objective function in the following three forms: material transportation carbon emission tax (Cmtrctax), refinery operation carbon emission tax (Croctax), and product transportation carbon emission tax (Cptrctax).

TAXC is the tax per ton of emitted, and can take the value of 10 Chinese Yuan per ton in the Chinese market [20]. CCOEF refers to carbon emission coefficient of different transportation modes. According to The Greenhouse Gas Protocol and the European Chemical Industry Association [24], the carbon emission coefficients of road, railway, water for freight are , respectively. The pipeline transportation is approximated as non-carbon emission. is the quantity of emitted per ton of material operated at refinery , readers interested in the calculation process can refer to the work of Jiang [25].

**The refinery operation of scenario n:**

where refers to the operation unit price [20] of refinery to process material at time period and refers to the quantity of material operated by refinery at time period of scenario .

The inventory of scenario :

where , and refer to the inventory unit price at time period for material in refinery , for product in terminal , , and refer to the quantity of material stocked at refinery , quantity of product stocked at terminal , and quantity of product stocked at distribution , respectively, at time period of scenario .

**The product transportation of scenario n**:

where refers to the transportation unit price for product from node to by transportation tool tr at time period refers to the distance between nodes and , as mentioned above; and refers to the quantity of product transported by transportation tool from node to at of scenario .

**The extra product purchase of scenario n:**

where refers to the unit price of extra product purchased at terminal at time period and refers to quantity of extra product purchased at terminal te at time period of scenario .

The production surplus penalty of scenario :

where refers to the penalty for production surplus per ton of product produced and and refer to the quantity of surplus product to be purchased at time period of by domestic customer , and overseas customer , respectively. The surplus penalty accounts for all the cost of coupons and discounts in order to sell the products to domestic and overseas customers.

The production backlog penalty of scenario n:

where refers to the penalty for production backlog per ton of product not being able to be produced and and refer to the quantity of backlog product needed at time period of scenario by domestic customer , and by overseas customer oc, respectively. The backlog penalty accounts for all the penalty for not being able to satisfy the amounts of products as signed in the contracts.  
**3.2. Constraints equations**

The variations of the variables are subjected to the following constraints.

Material balance:

where refers to the quantity of material stocked at refinery at time period of scenario .

**Product balance:**

where refers to the yield ratio of material to product at refinery and and refer to the quantity of product produced by refinery sold at of scenario at terminal , and at distribution , respectively.

The product balance constraint at terminal is given by

where refers to the quantity of extra product purchased at te to be sold at distribution at time period of scenario .

Function (18) gives the product balance constraint at distribution base.

**Sulfur content restriction**  
This constraint gives the logical relations of the sulfur content between the processed materials and the corresponding products, and would control the procurement for materials with different qualities.

where and refer to the sulfur content at time period of material and of product , respectively and refers to the desulfurization ratio of material at refinery .

**Procurement capacity limits**

where refers to the purchase upper limit of material at time period .

For the extra product procurement capacity, we have

where refers to the purchase upper limit of extra product at time period .

**Refinery operation**

where and refer to the lower and upper limits of operation capacity of refinery to process material at time period , respectively.

**Inventory capacity upper limits**

where refers to the inventory upper limit of refinery for material at time period and refer to inventory upper limit for product at time period in terminal , and in distribution base , respectively.

**Capacity for mixed flows**  
This constraint limits the maximum amount of flows transported by transportation tool between nodes involved at time period .  
where and refer to the flow of material , and of product , respectively, from node to by transportation tool at time period and refers to the transportation upper limit from node to by transportation tool at time period .

A set of constraints as follows gives the logical relations of mixed flows at each node.

,

**Product demands**  
This set of constraints denotes the demand constraint of oversea and domestic customers respectively.

where and , tp refer to the demand for product at time period of scenario of oversea customer oc and of domestic customer , respectively; and refer to the quantity of surplus product to be purchased at of by overseas customer and 'by domestic customer , respectively; and and , tp refer to the quantity of backlog product needed 'by oversea customer ',  
and 'by domestic customer ', respectively, at time period of scenario .

The upper bounds of each kind of product surplus or backlog are given as follows.  
where are variables, with values of 1 and 0 denoting the situation of happening and not happening of the corresponding incident as above, respectively; and refer to the upper limit of surplus product to be purchased at time period 'by oversea customer ', and 'by domestic customer , respectively; and and refer to the upper limit of backlog product needed at time period 'by oversea customer ', and 'by domestic customer ', respectively.

Since the surplus and backlog of one kind of product cannot happen at the same time period, the constraints as follows give the logical relationship.  
4. **Case Study**

The numerical data in the case study were collected both from real petrochemical market and secondary research like historical data, and the scale of this numerical example can be regarded as the same magnitude of a real world problem.

The time horizon in this study is one season and is divided into 3 time periods ( , each period accounts for one month. Materials include 2 kinds of crude oil (cof1, cof2) provided by the oil field, 6 kinds of crude oil (cote1, cote2, cote3, cote4, cote5, cote6) and 3 other materials (l-naphtha, h-naphtha, mix-xylene) provided by the terminal, the 11 kinds of materials mentioned above differ in their sulfur content and prices. There are 15 kinds of different products being produced by the three refineries, with 9 kinds of the products targeted at both oversea and domestic customers while other 6 products targeted only at domestic customers. The transportation network involves 4 kinds of transportation modes: ocean, pipeline, railway, and road transportation.  
**4.1. Scenario generation for deterministic model**

The first step before diving into the task of scenario generation was to calculate the determinist model as the contrast group. The deterministic model was defined as an exceptional case where the prices and demands of products remain constant, as and . Table 1 presents the results for the deterministic model.

Table 1 shows that the profit for the deterministic model is 6.42 USD (noted as profit ), and this would be the benchmark for the following groups with more scenarios. The deterministic model required a much lower computational effort (with an execution time of 0.078 s ), but it cannot capture the truly dynamic and uncertain behavior of the stochastic problem.

Table 1  
Deterministic model: Optimal profit and costs

|  |  |
| --- | --- |
| Item | Value |
| Profit | 6.4204611 |
| Sales | 15.573179 |
| Materials purchase cost | 6.1126469 |
| Transportation cost | 0.8806403 |
| Operations cost | 1.3364521 |
| Inventory cost | 0.0209382 |
| Extra products purchase cost | 0.5196594 |
| CO 2 tax cost | 0.0585877 |
| Surplus cost | 0.0529190 |
| Backorder cost | 0.1708746 |

4.2. Generation of more scenarios

As was mentioned in Section 2, the stochastic parameters in this case study were defined as the prices and demands of the products, and their probability distributions all follow the pattern of normal distribution, with the means and standard deviation being able to be forecasted. For the better illustration of the problem, we take one of the products as an example and denote the mean of this product as and the standard deviation as , and the same for the product demand: and .

Considering the symmetry of the normal distribution, using as the discretization unit can divide the whole probability space of the product price into parts, and for a case of , probabilities of each part are specified in Table 2. As predefined in Section 1, the random variables involved in this study are independent, so the demand component of the stochastic vector can be treated in a likely manner with the probability space of the product demand being divided into 5 parts as well. When the two components of the parameter vector were combined, 25 scenarios were generated. Other products are processed in the same manner.

A series of scenario generation with discretization index of , and can be proceeded in a similar fashion, and the sketched map of the 5 kinds of discretization for respectively , 11, 15, 23, and 29 is shown in Fig. 2.

The five sets of the aforementioned parameter discretization were fed into the original model to substitute the deterministic pattern, and statistics on the GAMS model with different numbers of scenarios are presented in Table 3.

Fig. 3 suggests that the discretization of the values of the uncertain parameters tends to overestimate the expected profit. When the discretization is increasingly refined (value of getting higher), the expected profit approaches the real value (the one with the continuous distributions). On the other hand, as the number of scenarios increased, the execution of the program required more computational efforts, denoted by the increase of elapsing time of the computation.

Facing the contradiction shown in Fig. 3, a proper value of needs to be determined to balance the calculation inaccuracy and the computational effort. Fig. 4 gives the trend of cumulative probability (probability of scenarios with profit greater than profit are counted into the cumulative probability) with respect to , it can be seen that the overall trend goes flat when it passes ( 225 scenarios), which indicates that scenarios more than 225 would achieve almost no more benefit.

Table 2  
Parameter discretization of price ( )

|  |  |  |
| --- | --- | --- |
| Scenario | Product price | Probability |
| 1 |  | 2.28 |
| 2 |  | 13.59 |
| 3 |  | 68.26 |
| 4 |  | 13.59 |
| 5 |  | 2.28 |

(e)  
(b)  
(d)

Fig. 2. Sketch map for stochastic parameter discretization.

As a result, the trends presented in Figs. 3 and 4 jointly pinpointed as the compromise of computational burden and accuracy of measurement. For a general problem, the ratio of the slope around an value can be calculated and used as a criterion to determine the desirable value. For an value in the th trial, namely , the criterion can be defined as  
In the particular example shown above when (while ), , which can be known as sufficiently small and can also be suggested for general applications. For more general cases, can be regarded as the criterion for determining .

After pinpointing the optimal value of as 15 for the case study, the distribution of the profits of the 225 scenarios should be explored. Fig. 5 gives the distribution of profits with respect to different product price and demand. The red curved surface is constituted by 225 dots, obviously the number of determines the smoothness of the surface. If the value of is too small, the surface would appear jagged, while when the value of approaches infinite, the surface would be close to the situation when the stochastic variables (product price and demand) in the problem are continuous. The blue plane with profit of USD (profit ) divides the whole space into 2 parts, and the dots above the blue plane represent scenarios with profit larger than profit .

Table 3  
Statistics of the GAMS model with uncertainty

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| |  | | --- | | Number of | | scenarios | | 25 | 121 | 225 | 529 | 841 |
| Blocks of equations | 91 | 91 | 91 | 91 | 91 |
| Single equations | 123,565 | 596,461 |  |  |  |
| Blocks of variables | 52 | 52 | 52 | 52 | 52 |
| Single variables | 158,626 | 764,290 |  |  |  |
| Nonzero elements |  |  |  |  |  |
| Discrete variables | 13,950 | 67,518 | 125,550 | 295,182 | 469,278 |

Fig. 3. Expected profits and elapsing time with respect to .

**4.3. Global sensitivity analysis**

In order to investigate the contributions of the uncertainty of each cost component on the output of the model, sensitivity analysis needs to be conducted. Local sensitivity analysis (LSA) method was frequently carried out to pinpoint the influential factors in different echelons of the petrochemical supply chain model [26-28], but it can only assess one single factor at a time by keeping all the other input parameters at their nominal values [29,30]. While the global sensitivity analysis (GSA) takes the full rank of uncertainties of the input variables, and all the uncertainty factors can be simultaneously evaluated. In this paper, the GSA method of Sobol'-Jansen (variance-based method) is applied to identify the influential cost components. The following steps are conducted in the GSA method:

**Step1: determine the objective function.**

The objective function of this problem can be rewritten as:

where Ctr is the sum of Cmtr and Cptr.

Fig. 4. Cumulative probability of scenarios with respect to .

Fig. 5. Distribution of optimal profits .

Step2: determine the input variables of the model.  
Since the target of the sensitivity analysis is to determine the influence of the 8 cost components on the overall profit, each cost component is regarded as an input variable and is assigned a coefficient ( .

Step3: generate samples from the distribution of input variables  
Considering the nature of the cost component in the economic context, the takes the value from 0.5 to 2.0 (denoting that the corresponding cost component takes the value from twice to half of its original value), and is assumed to follow the uniform distribution. The Monte Carlo method is applied, and 200 samples of the input variables are generated.

Step4: perform the sensitivity analysis  
When the goal is to determine the most important input variable, the first order sensitivity index is calculated, that is,  
The total sensitivity index indicates the magnitude of the interactions between the cost component and other components,

For details of this calculation process please refer to the work of Lilburne and Tarantola [33]. The interpretation of the index is straightforward, the higher the first order sensitivity index of an input variable, the greater their influence on the model output. While the lower the total sensitivity index , the less the interactions between cost component and other components. Fig. 6 gives the results of the GSA method.

The black dots in Fig. 6 show that the material purchase cost (Cmp), material and product transportation cost (Ctr), and refinery operation cost (Cro) rank as the top three influential cost components for the

Fig. 6. Results of the global sensitivity analysis.  
overall profit. While from the red dots in Fig. 6, one can notice that Cro exhibits greater interactions with other components when compared with the Ctr, regardless of the higher influence of Ctr on the overall profit.  
**5. Conclusions**

This work introduced a novel stochastic scenario generation method based on the probability distribution pattern of the stochastic parameters, and these scenarios were incorporated into the MILP model in order to capture the random factors in the stochastic optimization of the large-scale petrochemical supply chain. The scenario generation method showed good performance in portraying the stochastic characteristics of the petrochemical supply chain optimization problem, and the 225 discretized scenarios can be adequate enough in dealing with the continuous random parameters in the case study.

The best kick-point where computational cost and accuracy of approximations can be compromised was pinpointed, so that conclusions can be drawn as that for the scenario-based stochastic programming, reasonable numbers of discretized scenarios can be good approximations of the continuous random parameters. A criterion was established to determine the desirable value for general cases. In addition, the global sensitivity analyses gave the sequence of cost components' influence on the overall profit and the interactions of each cost component with other components.

For future study, in order to further reduce number of variables, reliable correlations between the random parameters may be incorporated into the model.

**Nomenclature**

|  |  |
| --- | --- |
| B | set of distribution bases indexed by b1, b2, b3 |
|  | penalty for production backlog per ton of product not being able to be produced at time period |
| C | set of domestic customers indexed by c1, c2, c3, c4, c5 |
|  | lower limit of operation capacity of refinery to process material at time period |
|  | upper limit of operation capacity of refinery to process material at time period |
|  | cost of production backlog penalty of scenario |
|  | cost of carbon emission tax of scenario |
|  | cost of extra product purchased of scenario |
|  | cost of inventory of scenario |
| Cmp | cost of material purchase |
| Cmtr | cost of material transportation |
|  | cost of product transportation of scenario |
|  | cost of refinery operation of scenario |

|  |  |
| --- | --- |
|  | cost of production surplus penalty of scenario |
|  | demand for product of oversea customer at time period of scenario |
|  | demand for product of domestic customer at time period of scenario |
|  | distance between node and |
|  | desulfurization ratio of material at refinery |
| ECr | quantity of emitted per ton of material operated at refinery |
| EPPU | purchase upper limit of extra product at time period |
| E[profit] | expected profit |

|  |  |
| --- | --- |
|  | unit price of extra product purchased at terminal at time period |
|  | inventory upper limit of refinery for material at time period |
|  | inventory upper limit of terminal for product at time period |
|  | inventory upper limit of distribution base for product at time period |
|  | inventory unit price for material at refinery at time period |
|  | inventory unit price for product at terminal at time period |
|  | inventory unit price for product at distribution base at time period |
|  | are variables |
|  | set of materials indexed by |
|  |  |
|  | of unit price of material purchased at oil field at time period |
|  | unit price of material purchased at terminal at time period |
|  | unit price of transportation for material by transportation tool at time period |
|  | purchase upper limit of material at time period |
|  | flow of material from node to by transportation tool at time period |
|  | set of nodes in the supply chain model indexed by and ) |
|  | set of oversea customers indexed by oc1, oc2 |
| OF | set of oil field indexed by of |
|  | set of products indexed by |
|  | probability of scenario |
| PTUP | transportation unit price for product from node to by transportation tool at time period |
|  | unit price for product purchased at terminal at time period of scenario |
|  | unit price for product purchased at distribution at time period of scenario |
|  | flow of product from refinery to distribution by transportation tool at time period of scenario |
|  | flow of product from refinery to terminal by transportation tool at time period of scenario |
|  | flow of product from distribution to domestic customer by transportation tool at time period of scenario |
|  | flow of product from terminal to overseas customer by transportation tool at time period of scenario |
|  | flow of product from terminal to distribution by transportation tool at time period of scenario |
|  | flow of product from node to by transportation tool at time period |
|  | upper limit of backlog product needed by oversea customer at time period |
|  | upper limit of backlog product needed by domestic customer at time period |
|  | upper limit of surplus product to be purchased by domestic customer at time period |
|  | upper limit of surplus product to be purchased by oversea customer at time period |
|  | quantity of backlog product needed by customer at time period of scenario |
|  | quantity of backlog product needed by oversea customer at time period of scenario |
|  | quantity of extra product purchased at terminal to be sold at distribution at time period of scenario |
|  | quantity of extra product purchased at terminal to be sold onsite at time period of scenario |
|  | quantity of material operated by refinery at time period of scenario |
|  | quantity of material purchased at terminal by refinery at time period |
|  | quantity of material purchased at oil field of by refinery at time period |
|  | quantity of material stocked at refinery at time period of scenario |
|  | quantity of material transported from node to by transportation tool at time period |
|  | quantity of product produced by refinery sold at distribution at time period of scenario |
|  | quantity of product sold at terminal to overseas customer at time period of scenario |
|  | quantity of product sold at distribution to domestic customer at time period of scenario |
|  |  |
|  | quantity of product stocked at distribution at time period of scenario |
|  | quantity of product stocked at terminal at time period of scenario |
|  | quantity of product produced by refinery sold at terminal at time period of scenario |
|  | quantity of product transported by transportation tool from node to at time period of scenario |
|  | quantity of surplus product to be purchased by customer at time period of scenario |
|  | quantity of surplus product to be purchased by customer at time period of scenario |
|  | set of refineries indexed by |
|  | operation unit price for refinery to process material at time period |
| Sales | sales of scenario |
|  |  |
|  | set of scenarios indexed by |
|  | sulfur content of material at time period |
|  | sulfur content of product at time period |
|  | penalty for production surplus per ton of product produced at time period |
| TAXC | tax per ton of emitted |
|  | transportation upper limit from node to by transportation tool at time period |
|  |  |
|  | set of terminals indexed by |
|  | set of time periods indexed by 1, 2, 3 |
|  | set of transportation tools indexed by 1, |
|  | yield ratio of material to product at refinery |